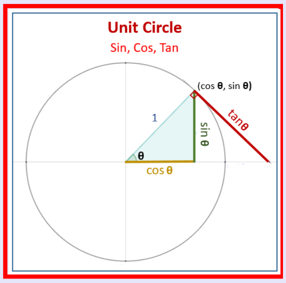
Math 241

Winter 2023

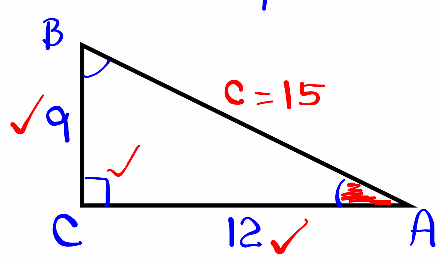
Lecture 4



Class QZ 3

Use the right triangle below to complete the following chart.

$\sin A = \frac{3}{5}$ ✓	$\csc A = \frac{5}{3}$ ✓
$\cos A = \frac{4}{5}$ ✓	$\sec A = \frac{5}{4}$ ✓
$\tan A = \frac{3}{4}$ ✓	$\cot A = \frac{4}{3}$ ✓



$c^2 = 9^2 + 12^2$

$c^2 = 225$ $\boxed{c = 15}$ ✓

Some algebra review:

1) Reduce $\dfrac{9xy}{3x-6y} = \dfrac{\cancel{9}^{3}xy}{\cancel{3}(x-2y)} = \dfrac{3xy}{x-2y}$

2) Simplify $\dfrac{2x^2+8x}{x^2-16} = \dfrac{2x\cancel{(x+4)}}{\cancel{(x+4)}(x-4)} = \dfrac{2x}{x-4}$

$x^2 - 4^2$

3) Reduce $\dfrac{x^2+2x-15}{x^2+x-12} = \dfrac{(x+5)\cancel{(x-3)}}{(x+4)\cancel{(x-3)}} = \dfrac{x+5}{x+4}$

4) Simplify $\dfrac{x^3-16x}{x^3+6x^2+8x} = \dfrac{\cancel{x}(x^2-16)}{\cancel{x}(x^2+6x+8)}$

$= \dfrac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(x+2)} = \boxed{\dfrac{x-4}{x+2}}$

5) Multiply: $\dfrac{2x}{3x-9} \cdot \dfrac{5x-15}{20x^3}$

$= \dfrac{\cancel{2x}}{3\cancel{(x-3)}} \cdot \dfrac{\cancel{5}^{1}\cancel{(x-3)}}{\cancel{2x}\cdot\cancel{10}_{2}x^2} = \dfrac{1}{3\cdot 2x^2}$

$= \boxed{\dfrac{1}{6x^2}}$

6) Simplify: $\dfrac{m^2-25}{20m} \cdot \dfrac{30m+10}{3m^2+16m+5}$

$= \dfrac{\cancel{(m+5)}(m-5)}{\cancel{10}\cdot 2m} \cdot \dfrac{\cancel{10}\cancel{(3m+1)}}{\cancel{(3m+1)}\cancel{(m+5)}} = \boxed{\dfrac{m-5}{2m}}$

7) Divide: $\dfrac{2x-10}{15x^3} \div \dfrac{x^2-5x}{12x}$

$= \dfrac{2x-10}{15x^3} \cdot \dfrac{12x}{x^2-5x} = \dfrac{2\cancel{(x-5)}}{\cancel{3x}\cdot 5x^2} \cdot \dfrac{4\cdot \cancel{3x}}{x\cancel{(x-5)}}$

$= \boxed{\dfrac{8}{5x^3}}$

8) Divide: $\dfrac{3x^2+10x+8}{18-6x} \div \dfrac{10+5x}{4x^3-12x^2}$

$= \dfrac{3x^2+10x+8}{18-6x} \cdot \dfrac{4x^3-12x^2}{10+5x} = \dfrac{(3x+4)\cancel{(x+2)}}{-\cancel{6}^{3}\cancel{(x-3)}} \cdot \dfrac{\cancel{4}^{2}x^2\cancel{(x-3)}}{5\cancel{(x+2)}}$

$-6x+18$

$= \boxed{\dfrac{2x^2(3x+4)}{-15}}$

$3x^2+10x+8 = \underbrace{3x^2+4x}+\underbrace{6x+8}$

24

Product = 24

Sum = 10

1, 24

2, 12

3, 8

$\boxed{4, 6}$

$= x(3x+4)+2(3x+4)$

$= (3x+4)(x+2)$

9) Simplify $\dfrac{x^2+2x}{x-7} + \dfrac{21-12x}{x-7}$

$= \dfrac{x^2+2x+21-12x}{x-7} = \dfrac{x^2-10x+21}{x-7}$

$= \dfrac{(x-3)\cancel{(x-7)}}{\cancel{x-7}} = \boxed{x-3}$

10) Reduce $\dfrac{2x^2+x-3}{x^2+6x+5} + \dfrac{x^2-2x+4}{x^2+6x+5} - \dfrac{2x^2+x+4}{x^2+6x+5}$

$= \dfrac{\cancel{2x^2}+\cancel{x}-3+x^2-2x+\cancel{4}-\cancel{2x^2}-\cancel{x}-\cancel{4}}{x^2+6x+5} = \dfrac{x^2-2x-3}{x^2+6x+5}$

$= \dfrac{\cancel{(x+1)}(x-3)}{(x+5)\cancel{(x+1)}} = \boxed{\dfrac{x-3}{x+5}}$

11) Simplify: $\dfrac{x}{x+2} - \dfrac{10}{x^2-x-6}$

$$= \frac{x(x-3)}{(x+2)(x-3)} - \frac{10}{(x-3)(x+2)}$$

$$= \frac{x^2-3x-10}{(x+2)(x-3)} = \frac{(x-5)\cancel{(x+2)}}{\cancel{(x+2)}(x-3)} = \frac{x-5}{x-3}$$

12) Reduce $\dfrac{x-7}{x^2+4x-5} - \dfrac{x-9}{x^2+3x-10}$

$$= \frac{(x-7)(x-2)}{(x+5)(x-1)(x-2)} - \frac{(x-9)(x-1)}{(x+5)(x-2)(x-1)}$$

$$= \frac{(x-7)(x-2) - (x-9)(x-1)}{(x+5)(x-1)(x-2)}$$

$$= \frac{\cancel{x^2} \cancel{-2x-7x} +14 \cancel{-x^2} +x +\cancel{9x} -9}{(x+5)(x-1)(x-2)} = \frac{\overset{1}{\cancel{x+5}}}{\cancel{(x+5)}(x-1)(x-2)} = \frac{1}{(x-1)(x-2)}$$

Solve: $\frac{2x}{3} + \frac{3x}{2} = \frac{13}{3}$

LCD = 6

$\cancel{6}^{2} \cdot \frac{2x}{\cancel{3}} + \cancel{6}^{3} \cdot \frac{3x}{\cancel{2}} = \cancel{6}^{3} \cdot \frac{13}{\cancel{2}}$

$4x + 9x = 39$

$13x = 39$

$x = \frac{39}{13}$

$\boxed{x = 3}$

Solution Set

$\{3\}$

Solve $3 + \frac{9-2x}{8x} = \frac{5}{2x}$

LCD = 8x

$8x \cdot 3 + \cancel{8x} \cdot \frac{9-2x}{\cancel{8x}} = \cancel{8x}^{4} \cdot \frac{5}{\cancel{2x}}$

$24x + 9 - 2x = 20$

$22x = 20 - 9$

$22x = 11 \quad x = \frac{11}{22}$

$\boxed{x = \frac{1}{2}}$ $\left\{\frac{1}{2}\right\}$

Solve $\frac{3}{x+4} - \frac{1}{x-4} = \frac{2}{x^2-16}$

LCD = $(x+4)(x-4)$

$\cancel{(x+4)}(x-4) \cdot \frac{3}{\cancel{x+4}} - (x+4)\cancel{(x-4)} \cdot \frac{1}{\cancel{x-4}} = \cancel{(x+4)(x-4)} \cdot \frac{2}{\cancel{x^2-16}}$

$3(x-4) - (x+4) = 2$

$3x - 12 - x - 4 = 2$

$2x = 2 + 16$

$2x = 18 \quad \boxed{x = 9}$

$\{9\}$

Solve

$$\frac{1}{y^2-5y+6}+\frac{y}{y^2-2y-3}=\frac{y+2}{y^2-y-2}$$

$$\frac{1}{(y-3)(y-2)}+\frac{y}{(y-3)(y+1)}=\frac{y+2}{(y-2)(y+1)}$$

$$LCD=(y-3)(y-2)(y+1)$$

$$1(y+1)+y(y-2)=(y+2)(y-3)$$

$$y+1+\cancel{y^2}-2y=\cancel{y^2}-3y+2y-6$$

$$\cancel{-y}+1=\cancel{-y}-6$$

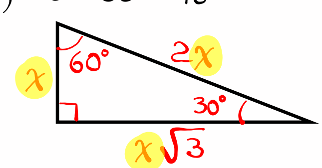
$$1=-6 \rightarrow \text{False}$$

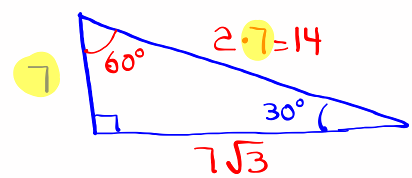
No Solution

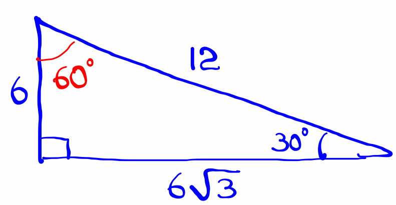
$\emptyset$

Special Right Triangles:

1) $30°-60°-90°$

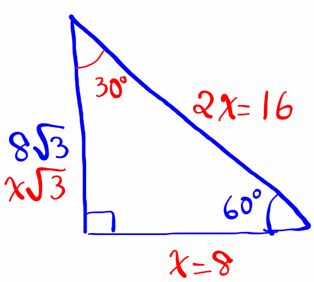




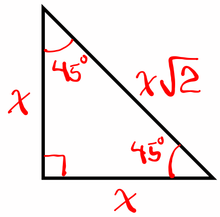


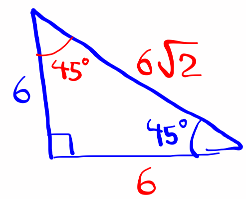
$12=2x$

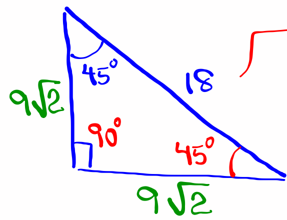
$x=6$



2) 45°-45°-90°

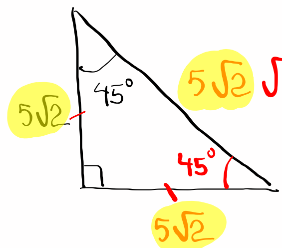






$18 = x\sqrt{2}$

$$x = \frac{18}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{18\sqrt{2}}{\sqrt{4}} = \frac{\overset{9}{\cancel{18}}\sqrt{2}}{\cancel{2}} = 9\sqrt{2}$$

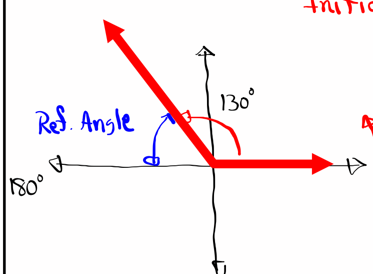


$5\sqrt{2}\,\sqrt{2} = 5\sqrt{4} = 5 \cdot 2 = 10$

Draw $\alpha = 130°$ in standard position, give its ref. angle.

Vertex at (0,0)

Initial Side on x-axis, $x>0$



Counterclockwise

R.A. = 180° − 130° = 50°

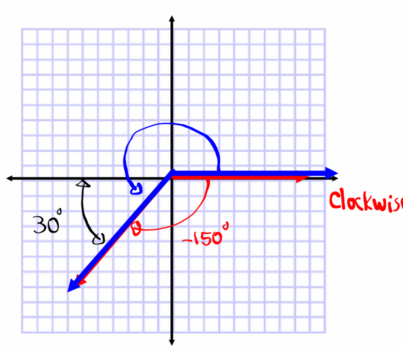
Draw $\alpha = -150°$ in standard position, give its Ref. Angle.

Coterminal angle

360° − 150° = 210°

R.A. = 210° − 180°

= 30°



Draw $\alpha = \frac{4\pi}{3}$ in standard position, give its ref. angle in radians.

$\frac{4\pi}{3}$ Rad $= \frac{4(\cancel{180°})^{60°}}{\cancel{3}}$

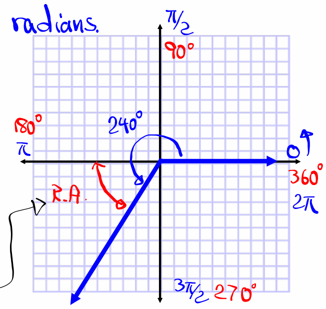
$= 4(60°)$

$= 240°$

R.A. $= \frac{4\pi}{3} - \pi$

$= \frac{4\pi}{3} - \frac{3\pi}{3} = \frac{\pi}{3}$

$= 60°$



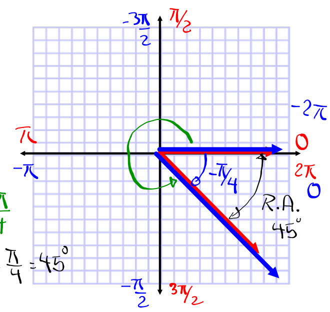
Draw $\alpha = -\frac{\pi}{4}$ in standard position, find a positive angle that is coterminal with α, find its ref. angle in degrees.

$\frac{\pi}{4} = \frac{180°}{4} = 45°$

$2\pi - \frac{\pi}{4} = \boxed{\frac{7\pi}{4}}$

$\frac{2\pi \cdot 4}{4} - \frac{\pi}{4} = \frac{8\pi}{4} - \frac{\pi}{4} = \frac{7\pi}{4}$

R.A. $= 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4} = 45°$



Verify $\frac{\cos^2 x}{1 + \sin x} = 1 - \sin x$ ✓

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$A^2 - B^2 = (A-B)(A+B)$

$$\frac{\cos^2 x}{1 + \sin x} = \frac{1 - \sin^2 x}{1 + \sin x}$$

$$= \frac{(1 - \sin x)\cancel{(1 + \sin x)}}{\cancel{1 + \sin x}} = 1 - \sin x \checkmark$$

Verify $(\sec x - \tan x)^2 = \dfrac{1-\sin x}{1+\sin x}$ ✓

$$(\sec x - \tan x)^2 = \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2$$

$$= \left(\frac{1-\sin x}{\cos x}\right)^2 = \frac{(1-\sin x)(1-\sin x)}{\cos^2 x}$$

$$= \frac{(1-\sin x)(1-\sin x)}{1-\sin^2 x}$$

$$= \frac{\cancel{(1-\sin x)}(1-\sin x)}{\cancel{(1-\sin x)}(1+\sin x)}$$

$$\frac{1-\sin x}{1+\sin x}$$

Verify ✓ $\sec x + \tan x = \dfrac{1}{\sec x - \tan x}$

$A^2 - B^2$

$$\frac{1}{\sec x - \tan x} = \frac{\sec^2 x - \tan^2 x}{\sec x - \tan x}$$

$$= \frac{\cancel{(\sec x - \tan x)}(\sec x + \tan x)}{\cancel{\sec x - \tan x}}$$

$$= \sec x + \tan x$$

$\sin^2 x + \cos^2 x = 1$

$\boxed{1 + \tan^2 x = \sec^2 x}$

$1 + \cot^2 x = \csc^2 x$

$1 = \sec^2 x - \tan^2 x$

Verify $\dfrac{\cos^2 x \sin x - \cos^3 x}{2\sin^4 x - \sin^2 x} = \dfrac{\cot^2 x}{\sin x + \cos x}$ ✓

$$\frac{\cos^2 x\,(\sin x - \cos x)}{\sin^2 x\,(2\sin^2 x - 1)} = \frac{\cot^2 x\,(\sin x - \cos x)}{2\sin^2 x - 1}$$

$$= \frac{\cot^2 x\,(\sin x - \cos x)}{2\sin^2 x - (\sin^2 x + \cos^2 x)}$$

SG 5 ✓

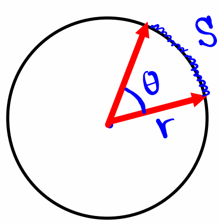
$$= \frac{\cot^2 x\,(\sin x - \cos x)}{2\sin^2 x - \sin^2 x - \cos^2 x}$$

$$= \frac{\cot^2 x\,(\sin x - \cos x)}{\sin^2 x - \cos^2 x}$$

$A^2 - B^2$

$$= \frac{\cot^2 x\,\cancel{(\sin x - \cos x)}}{(\sin x + \cos x)\cancel{(\sin x - \cos x)}} = \frac{\cot^2 x}{\sin x + \cos x}$$ ✓

Consider a circle with radius r, make an angle with vertex at the center with measure θ radian.



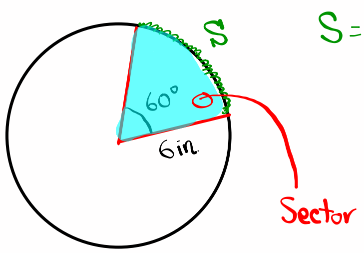
$S = r\theta$

If $S = r \Rightarrow \theta = 1$ Radian

Suppose $r = 4$ cm, $\theta = \frac{\pi}{4}$ Rad.

$S = r\theta = 4 \cdot \frac{\pi}{4} = \boxed{\pi \text{ cm}}$

Find S if $r = 6$ in., and $\theta = 60°$.

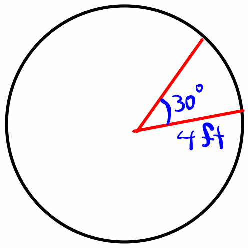


$S = r\theta$, θ must be in radians

$= 6 \cdot \frac{\pi}{3} = \boxed{2\pi \text{ in}}$

$60° = \frac{180°}{3} = \frac{\pi}{3}$ Rad.

Area of a Sector $A = \frac{1}{2}r^2\theta$, θ must be in radians.



$30° = \frac{180°}{6} = \frac{\pi}{6}$ Rad.

$A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \cdot 4^2 \cdot \frac{\pi}{6}$

$= \frac{1}{\cancel{2}} \cdot \cancel{16}^{\,8\ 4} \cdot \frac{\pi}{\cancel{6}_{3}} = \boxed{\frac{4\pi}{3}\ ft^2}$

Find area of a Sector with $r = 10cm$ and $\theta = 10°$.

$A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \cdot 10^2 \cdot \frac{\pi}{18}$

$= \frac{1}{\cancel{2}} \cdot \cancel{100}^{\,\cancel{50}\ 25} \cdot \frac{\pi}{\cancel{18}_{9}} = \boxed{\frac{25\pi}{9}\ cm^2}$

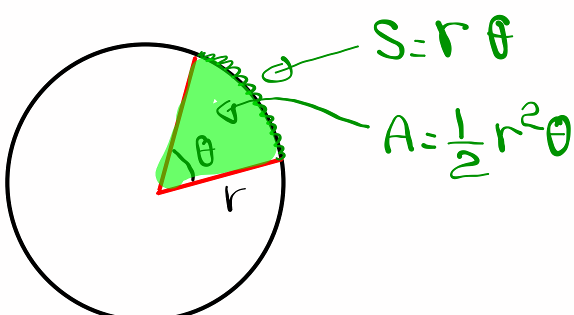
$10° =$

$180° = \pi$ Rad.

$1° = \frac{\pi}{180}$ Rad.

$10° = \frac{10\pi}{180} = \frac{\pi}{18}$

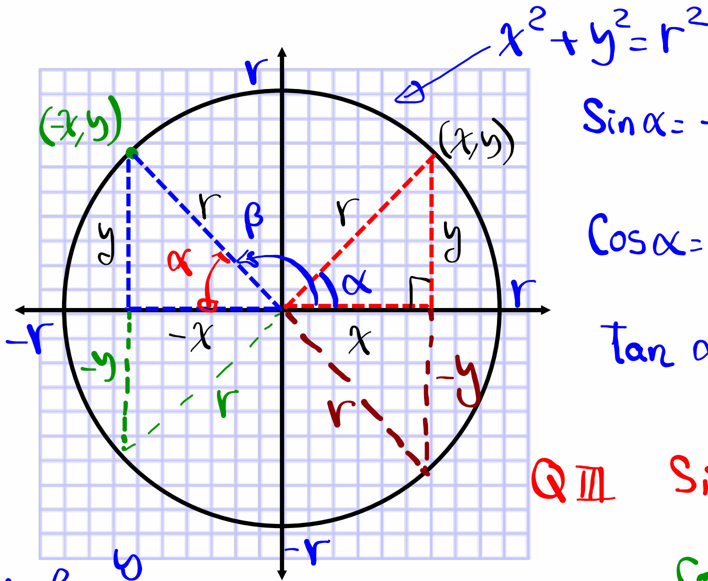
Sector of a Circle with radius r and Central angle θ in radians.



Use $\theta = 2\pi$, for any radius r, find

$S = r\theta = r \cdot 2\pi = 2\pi r$ Circumference of a Circle

$A = \frac{1}{2}r^2\theta = \frac{1}{\cancel{2}} \cdot r^2 \cdot \cancel{2}\pi = r^2\pi = \pi r^2$ Area of a Circle



$\sin\alpha = \frac{y}{r}$

$\cos\alpha = \frac{x}{r}$

$\tan\alpha = \frac{y}{x}$

Q III

$\sin A = \frac{-y}{r}$

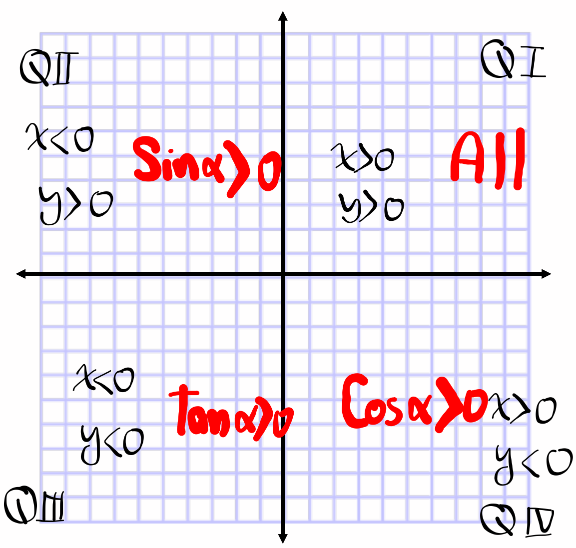
$\cos A = \frac{-x}{r}$

$\tan A = \frac{-y}{-x} = \frac{y}{x}$

$\sin\beta = \frac{y}{r}$

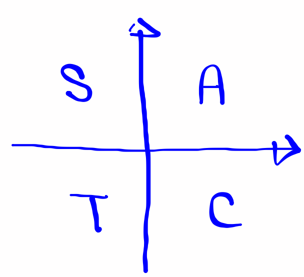
$\cos\beta = \frac{-x}{r}$

$\tan\beta = \frac{y}{-x}$

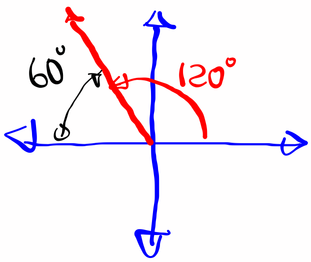


$\left.\begin{array}{l} \sin\alpha = \frac{y}{r} \\ \cos\alpha = \frac{x}{r} \\ \tan\alpha = \frac{y}{x} \end{array}\right\} r>0$

α is the ref. Angle.



Find $\sin 120°$, $\cos 120°$, and $\tan 120°$.

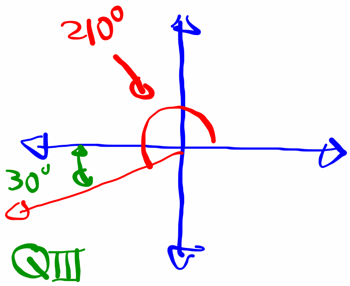


$\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos 120° = -\cos 60° = -\frac{1}{2}$

$\tan 120° = -\tan 60° = -\sqrt{3}$

Find $\sin 210°$, $\cos 210°$, and $\tan 210°$.



$\sin 210° = -\sin 30° = -\frac{1}{2}$

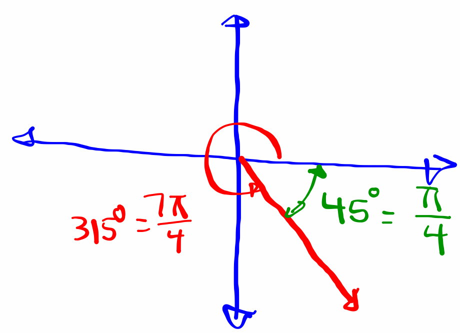
$\cos 210° = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\tan 210° = \tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Find $\sin \frac{7\pi}{4}$, $\cos \frac{7\pi}{4}$, and $\tan \frac{7\pi}{4}$.

$\frac{7\pi}{4} = \frac{7(180°)}{4} = 315°$

Q IV

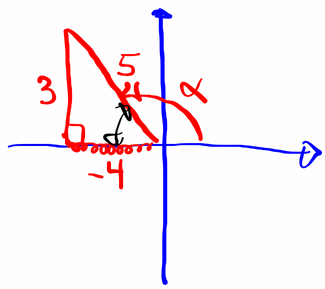


$\sin \frac{7\pi}{4} = -\sin \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\cos \frac{7\pi}{4} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\tan \frac{7\pi}{4} = -\tan \frac{\pi}{4} = -1$

Given $\sin\alpha = \frac{3}{5}$, $90^\circ < \alpha < 180^\circ$

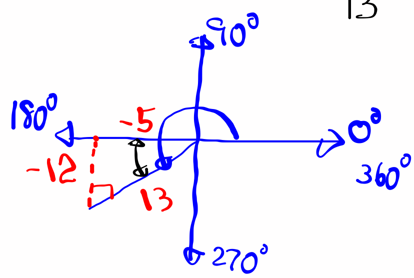


$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{-4}{5}$

$\tan\alpha = \frac{3}{-4} = \frac{-3}{4}$

Given $\cos\alpha = \frac{-5}{13}$, $180^\circ < \alpha < 270^\circ$



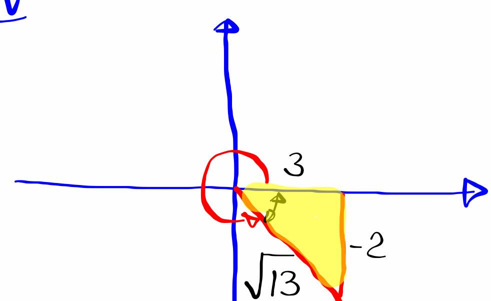
$\sin\alpha = \frac{-12}{13}$

$\cos\alpha = \frac{-5}{13}$

$\tan\alpha = \frac{-12}{-5} = \frac{12}{5}$

$\tan\alpha = \frac{-2}{3}$, α in QIV

$\tan\alpha = \frac{y}{x} = \frac{-2}{3}$

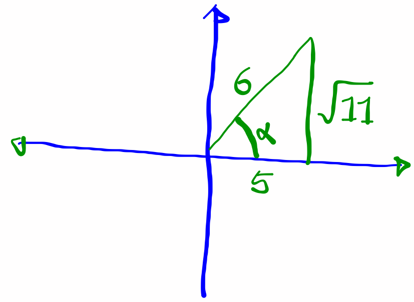


$\sin\alpha = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$

$\cos\alpha = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$

$\tan\alpha = \frac{-2}{3}$

$\cos\alpha = \frac{5}{6}$, $\underbrace{0^\circ < \alpha < 90^\circ}_{QI}$



$\sin\alpha = \frac{\sqrt{11}}{6}$

$\cos\alpha = \frac{5}{6}$

$\tan\alpha = \frac{\sqrt{11}}{5}$

Two angles are Complementary if their Sum $= 90^\circ$

" " " Supplementary " " " $= 180^\circ$

Find the Complement and Supplement of $\alpha = 15^\circ$.

Comp. $+ 15^\circ = 90^\circ \rightarrow$ Comp. $= 90^\circ - 15^\circ = \boxed{75^\circ}$

Supp. $+ 15^\circ = 180^\circ \rightarrow$ Supp. $= 180^\circ - 15^\circ = \boxed{165^\circ}$

Find the Complement and Supplement for $\alpha = \frac{\pi}{5}$ Rad.

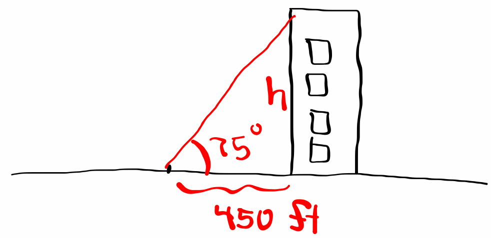
Comp. $+ \frac{\pi}{5} = \frac{\pi}{2} \rightarrow$ Comp. $= \frac{\pi}{2} - \frac{\pi}{5} = \frac{5\pi - 2\pi}{10} = \boxed{\frac{3\pi}{10}}$

Supp. $+ \frac{\pi}{5} = \pi \rightarrow$ Supp. $= \pi - \frac{\pi}{5} = \boxed{\frac{4\pi}{5}}$

From 450 ft of a building, the angle of elevation to the top of the building is 75°.

How tall is the building?

$$\tan 75° = \frac{h}{450}$$



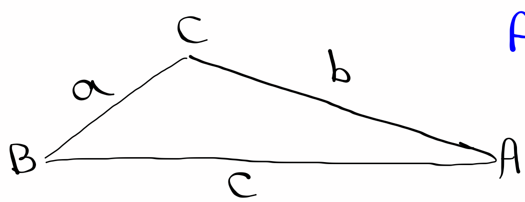
$$h = 450 \cdot \tan 75°$$

$$= 1679.422863$$

Round up

$$h \approx 1680 \text{ ft}$$

Heron's formula:



$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \dfrac{a+b+c}{2}$

$$s = \frac{3+7+8}{2} = \frac{18}{2} = 9$$

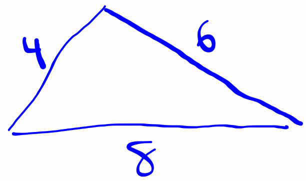
$$\text{Area} = \sqrt{9(9-3)(9-7)(9-8)}$$

$$= \sqrt{9 \cdot 6 \cdot 2 \cdot 1}$$

$$= \sqrt{108} \approx 10.392$$

$$\approx 10.4 \text{ units}^2$$

Find area of a triangle with sides 4 in, 6 in, and 8 in.



Using Heron's Formula

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2} = \frac{4+6+8}{2} = \frac{18}{2} = 9$$

$$Area = \sqrt{9(9-4)(9-6)(9-8)} = \sqrt{9 \cdot 5 \cdot 3 \cdot 1} = \sqrt{135}$$

$$= 11.618 \ldots$$

$$\approx 12 \text{ in}^2.$$